# VCE Circus Primer
# Linear Springs

## Basic Formulae:

Hooke's law, for a plain ideal linear spring: $F = - kx$ where:

$x$ is the deflection,
$k$ is the spring constant (a measure of stiffness) and
$F$ is the force producing the deflection.
The negative sign indicates that the deflection and force are in opposing directions.

The stored, or potential energy in a spring is given by $E_S = 1/2\ kx^2$.

## Energy Exchange

Questions about energy exchange and springs are very popular with Physics 3-4 examiners. Generally the relevant equations are:

| | |
|---|---|
| $E_{spring} = 1/2\ kx^2$ | potential energy stored in a spring |
| $E_{kinetic} = 1/2\ mv^2$ | kinetic energy in a moving mass |
| $P_G = mgh$ | energy stored in a mass because of its vertical position |

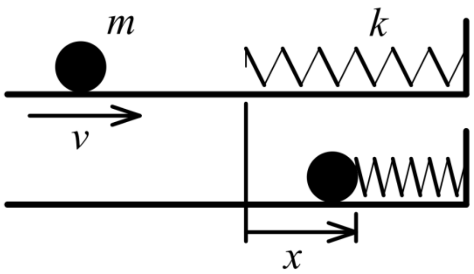
## Examples

1) A mass $m$ is moving with velocity $v$ is momentarily brought to rest by a spring with a spring constant $k$. The spring is compressed by distance $x$. What are the relationships between these four parameters?

Conservation of energy applies since the question does give any indication of losses.

So:

$$E_{spring} = E_{kinetic}$$




When the mass is at rest the spring has acquired the energy lost by the mass. That is

$$1/2\ kx^2 = 1/2\ mv^2$$

or

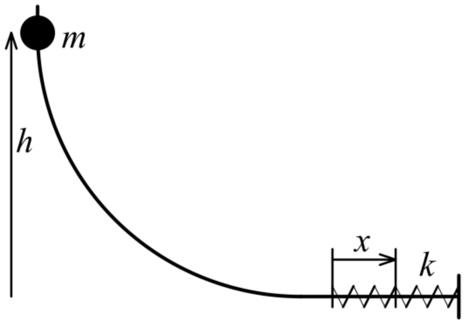
$$kx^2 = mv^2.$$

2) A bead of mass $m$ initially starts at height h and slides down the frictionless guide rod to contact a spring with a spring constant $k$. The bead is bought momentarily to rest after deflecting the spring by $x$.

Two energy conversions take place in this process – the potential energy of the elevated mass is converted to the kinetic energy of a moving mass as the bead descends and this kinetic energy is converted to the potential energy of a compressed spring as the mass comes to rest.

So:

$$E_{kinetic} = E_{elevated}$$
$$1/2\ mv^2 = mgh$$
$$v^2 = 2gh$$




and:

$$E_{spring} = E_{kinetic}$$
$$1/2\ kx^2 = 1/2\ mv^2$$

or

$$kx^2 = mv^2$$

Alternatively we can dispense with the conversion to and from kinetic energy and note that:

$$E_{spring} = E_{elevation}$$
$$1/2\ kx^2 = mgh$$

or

$$kx^2 = 2mgh$$

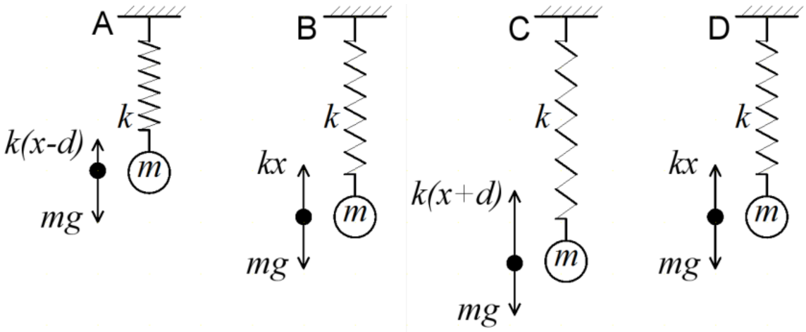
3) A mass oscillating while suspended on a spring is straightforward configuration but because of the three interacting energy exchanges can be confusing.

Here we look at four points in the motion of the mass:

A) At the upper limit of movement.
B) Passing through the equilibrium point on the way down.
C) At the lower limit of movement.
D) Passing through the equilibrium point on the way up.

| **location** | **extension** | **spring force** | **velocity** | **acceleration** | **spring PE** | **mass KE** | **mass PE** |
|---|---|---|---|---|---|---|---|
| A | $-d$ | $k(x-d)$ | *0* | *- maximum* | *minimum* | *0* | *maximum* |
| B | *0* | $k(x) = mg$ | *- maximum* | *0* | *neutral* | *maximum* | *neutral* |
| C | $+d$ | $k(x+d)$ | *0* | *+ maximum* | *maximum* | *0* | *minimum* |
| D | *0* | $k(x) = mg$ | *+ maximum* | *0* | *neutral* | *maximum* | *neutral* |

i) Weight force is constant at *mg*. Refer to FBDs below.
ii) Absolute value of mass PE is dependent on reference point.
iii) Equilibrium value is labelled as *neutral*.




Physics 3-4 examination questions involving springs.

2017 Section A, Questions 12, 13
2018 Section B, Question 6
2019 Section B, Question 19 (This question proved to be difficult for many students.)
2020 Section B, Question 9
2021 Section A, Question 12
2021 Section B, Question 9

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Comments? Spotted some errors? Email Harlan via logic@logicfronttoback.nz